1. This statement is false, according to the truth table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p | q | r | ¬(p ∧ q ∨ r) | ¬p ∨ ¬q ∧ ¬r |
| T | T | T | F | F |
| T | T | F | F | F |
| T | F | T | F | F |
| T | F | F | T | T |
| F | T | T | F | T |
| F | T | F | T | T |
| F | F | T | F | T |
| F | F | F | T | T |

1. In this problem, X can be any number in the set of real numbers.

Prove that |x| ≥ -x. Let a = -x.

* 1. Case 1: a < 0:

|a| == -a

|a| > 0 because x > 0 (according to the definition of absolute value)

|a| > 0 > a, therefore |a| > a.

* 1. Case 2: a ≥ 0:

|a| == a

1. Prove that for any two real numbers, x and y, |x + y| ≤ |x| + |y|.

We have established the definition of absolute value, |a| = -a if a< 0.

* 1. Case 1: b ≥ 0:

b = |x| + |y|

x ≥ 0, y ≥ 0

|x| = x, |y| = y

b = x + y, therefore b ≥ 0

* 1. Case 2: b < 0

b = |x + y|

x < 0, 0 < y < |x|

b = |b| = -b, according to the definition of absolute value

1. Proof by contrapositive: Prove that for any nonzero real number , if is irrational then is also irrational

**If is rational, then is a rational number. Since , must be a rational number.**

**.**

1. Proof by contradiction: Prove that the average of any set of real numbers {a1, a2, a2, …, an} is greater than or equal to at least one of the numbers.

Let’s suppose that for each number in the set (a, b, c) is less than the average of the three numbers.

Let = (a set of real numbers).

Let = the average defined by .

**Our assumption states that . If we combine these statements, we get , but is defined to be . Such, we get which is a contradiction.**

1. *a*. Statement: A property owner who received $15,000 of rent in 2019 must have collected $1250 or more during at least one month.

Proof technique: Proof by contrapositive

Assume that the property owner has not collected $1250 or more for all months of 2019.

Let a = {m1, m2, m3, …, m12} (the total amount of rent collected in 2019)

According to our assumption, for any value of a, that value is less than $1250.

**If every value of a is less than $1250, then a < $15,000. This is clearly contrapositive compared to our original statement, thus proving its validity.**

*b*. Statement: The sum of any four consecutive integers is even.

Proof technique: direct proof.

Let a = an integer that is the first the four consecutive integers. This number is either odd or even.

Therefore, , , , and are the four consecutive integers.

**The definition of the sum of these integers is which is a multiple of 2. Multiples of 2 are even, so thus the sum of the integers are even.**

*c.* Statement: Let . If is odd, then is odd and is odd.

Proof technique: proof by contradiction.

**Assume that and are both not odd, which means that one or the other might be even or both are even. As such, the result, is even which contradicts our statement.**

*d*. Statement: There is exactly one perfect number between 2 and 10, inclusive.

Proof technique: proof by cases.

*The definition of a perfect number simply means that it is a positive integer that is equal to the sum of its proper divisors.*

**Case 1 – In this case, assume our number is a composite number.**

Our composite number options are {4, 6, 8, 9, 10}

The only perfect number in that set is six.

Since there is only one perfect number, that proves the statement in this case.

**Case 2 – In this case, assume our number is a prime number.**

A prime number’s only proper divisor can only be 1.

Since 1 is not a perfect number, we can establish that no prime numbers can be perfect.

Let p be the set of prime numbers in our set of numbers between 2 and 10, inclusive.

p = {2, 3, 5, 7}, for all of p, there does not exist a perfect number.